Proportional and Nonproportional Relationships and Functions

MODULE 3

Proportional Relationships

FL   8.EE.2.5, 8.EE.2.6, 8.F.1.2, 8.F.2.4

MODULE 4

Nonproportional Relationships

FL   8.EE.2.6, 8.F.1.2, 8.F.1.3, 8.F.2.4

MODULE 5

Writing Linear Equations

FL   8.F.2.4, 8.SP.1.1, 8.SP.1.2, 8.SP.1.3

MODULE 6

Functions

FL   8.EE.2.5, 8.F.1.1, 8.F.1.2, 8.F.1.3, 8.F.2.4, 8.F.2.5

CAREERS IN MATH

Cost Estimator A cost estimator determines the cost of a product or project, which helps businesses decide whether or not to manufacture a product or build a structure. Cost estimators analyze the costs of labor, materials, and use of equipment, among other things. Cost estimators use math when they assemble and analyze data. If you are interested in a career as a cost estimator, you should study these mathematical subjects:

• Algebra
• Trigonometry
• Calculus

Research other careers that require analyzing costs.

Unit 2 Performance Task

At the end of the unit, check out how **cost estimators** use math.

Vocabulary Preview

Use the puzzle to preview key vocabulary from this unit. Unscramble the circled letters within the found words to answer the riddle at the bottom of the page.

```
D  I  N  T  F  O  K  F  Z  Y  X  I  R  H (L)
T  I  D (O) N  O  T  N  I  R  A  V  Z  B  I
U  U  R  Z  I  E  K  N  M  C  J  E  H  Z  N
(P) N  X  E  V  T  T  H  W  C  V  W  S  E  E
T  A  E  B  U  E  C  E  K  Q  O  P  D  U  A
U  I  V  A (R) N  P (N) M  K  Q  W  N  Z  R
(O) X  G  C  B  O  I  Q  U  M  Y  X  P  L  E
C  P  E  B (L) K  B  T  J  F  H  L  A  K  Q
J (P) V  S  L  Q  M  J (R) G  H  R  U  N  U
T  E  F  N  I  Z  B  M  C  A  D  A  X  A (A)
T  T  O  S  V  D  T  H  W  B  T  D  B  E  T
K  O  O  I  I  E  Y  W  U  O  Q  E  P  K  I
P  G  T  G  L  S  O  J  Q  R  F  D  I  Q (O)
D  T  A  M  H  Q  T  S  T  A  J  Q  X  O  N
B (I) V  A  R  I  A  T  E  D  A (T) A  Y  N
```

- The *y*-coordinate of the point where the graph crosses the *y*-axis. (**Lesson 4.2**)
- A rule that assigns exactly one output to each input. (**Lesson 6.1**)
- The result after applying the function machine's rule. (**Lesson 6.1**)
- A rate in which the second quantity in the comparison is one unit. (**Lesson 3.3**)
- The ratio of change in rise to the corresponding change in run on a graph. (**Lesson 3.2**)
- A set of data that is made up of two paired variables. (**Lesson 5.3**)
- An equation whose solutions form a straight line on a coordinate plane. (**Lesson 4.1**)

Q: How much of the money earned does a professional sports team pay its star athlete?

A: An ___ ___ ___ ___ ___ ___ – ___ ___ ___ ___ ___ ___ ___!

Proportional Relationships

? ESSENTIAL QUESTION

How can you use proportional relationships to solve real-world problems?

Real-World Video

Speedboats can travel at fast rates while sailboats travel more slowly. If you graphed distance versus time for both types of boats, you could tell by the steepness of the graph which boat was faster.

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Are YOU Ready?

Complete these exercises to review skills you will need for this module.

Write Fractions as Decimals

EXAMPLE $\dfrac{1.7}{2.5} = ?$

Multiply the numerator and the denominator by a power of 10 so that the denominator is a whole number.

$$\dfrac{1.7 \times 10}{2.5 \times 10} = \dfrac{17}{25}$$

Write the fraction as a division problem. Write a decimal point and zeros in the dividend.
Place a decimal point in the quotient.
Divide as with whole numbers.

$$\begin{array}{r} 0.68 \\ 25\overline{)17.00} \\ -15\,0 \\ \hline 2\,00 \\ -2\,00 \\ \hline 0 \end{array}$$

Write each fraction as a decimal.

1. $\dfrac{3}{8}$ _____

2. $\dfrac{0.3}{0.4}$ _____

3. $\dfrac{0.13}{0.2}$ _____

4. $\dfrac{0.39}{0.75}$ _____

5. $\dfrac{4}{5}$ _____

6. $\dfrac{0.1}{2}$ _____

7. $\dfrac{3.5}{14}$ _____

8. $\dfrac{7}{14}$ _____

9. $\dfrac{0.3}{10}$ _____

Solve Proportions

EXAMPLE $\dfrac{5}{7} = \dfrac{x}{14}$

$\dfrac{5 \times 2}{7 \times 2} = \dfrac{x}{14}$ $7 \times 2 = 14$, so multiply the numerator and denominator by 2.

$\dfrac{10}{14} = \dfrac{x}{14}$ $5 \times 2 = 10$

$x = 10$

Solve each proportion for x.

10. $\dfrac{20}{18} = \dfrac{10}{x}$ _____

11. $\dfrac{x}{12} = \dfrac{30}{72}$ _____

12. $\dfrac{x}{4} = \dfrac{4}{16}$ _____

13. $\dfrac{11}{x} = \dfrac{132}{120}$ _____

14. $\dfrac{36}{48} = \dfrac{x}{4}$ _____

15. $\dfrac{x}{9} = \dfrac{21}{27}$ _____
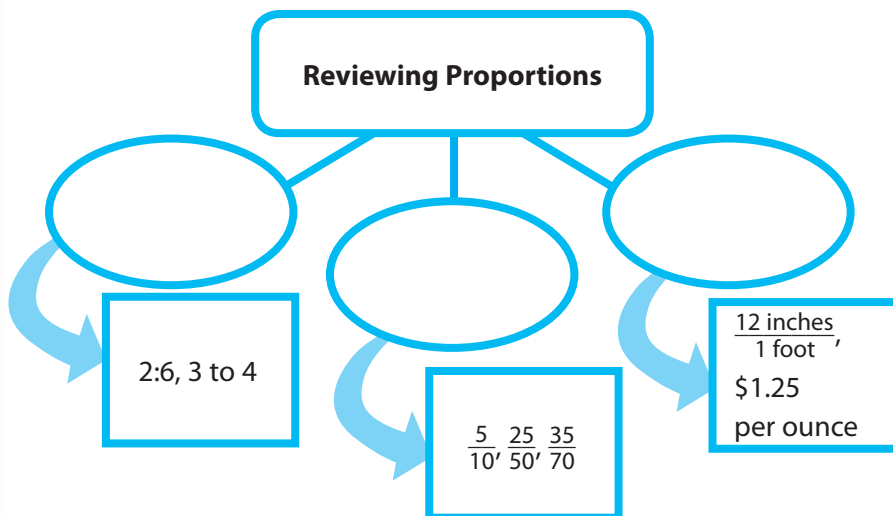
16. $\dfrac{24}{16} = \dfrac{x}{2}$ _____

17. $\dfrac{30}{15} = \dfrac{6}{x}$ _____

18. $\dfrac{3}{x} = \dfrac{18}{36}$ _____

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the diagram.

```
                    ┌─────────────────────────┐
                    │  Reviewing Proportions  │
                    └─────────────────────────┘
             ┌─────────────┼─────────────┐
         ◯             ◯             ◯
         │                              │
         ↓                              ↓
  ┌──────────────┐                ┌──────────────┐
  │  2:6, 3 to 4 │                │  12 inches   │
  │              │                │  ──────── ,  │
  └──────────────┘                │   1 foot     │
                  ↓               │  $1.25       │
           ┌──────────────┐       │  per ounce   │
           │  5   25   35  │      └──────────────┘
           │ ──  ──   ──   │
           │ 10  50   70   │
           └──────────────┘
```

Vocabulary

Review Words

constant *(constante)*
✔ equivalent ratios *(razones equivalentes)*
proportion *(proporción)*
rate *(tasa)*
✔ ratios *(razón)*
✔ unit rates *(tasas unitarias)*

Preview Words

constant of proportionality *(constante de proporcionalidad)*
proportional relationship *(relación proporcional)*
rate of change *(tasa de cambio)*
slope *(pendiente)*

Understand Vocabulary

Match the term on the left to the definition on the right.

1. unit rate

2. constant of proportionality

3. proportional relationship

A. A constant ratio of two variables related proportionally.

B. A rate in which the second quantity in the comparison is one unit.

C. A relationship between two quantities in which the ratio of one quantity to the other quantity is constant.

Active Reading

Key-Term Fold Before beginning the module, create a key-term fold to help you learn the vocabulary in this module. Write the highlighted vocabulary words on one side of the flap. Write the definition for each word on the other side of the flap. Use the key-term fold to quiz yourself on the definitions used in this module.

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

FL 8.EE.2.5

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

Key Vocabulary

proportional relationship
(relación proporcional)
A relationship between two quantities in which the ratio of one quantity to the other quantity is constant.

slope *(pendiente)*
A measure of the steepness of a line on a graph; the rise divided by the run.

unit rate *(tasa unitaria)*
A rate in which the second quantity in the comparison is one unit.

What It Means to You

You will use data from a table and a graph to apply your understanding of rates to analyzing real-world situations.

UNPACKING EXAMPLE 8.EE.2.5

The table shows the volume of water released by Hoover Dam over a certain period of time. Use the data to make a graph. Find the slope of the line and explain what it shows.

Water Released from Hoover Dam	
Time (s)	Volume of water (ft³)
5	75,000
10	150,000
15	225,000
20	300,000

Water Released from Hoover Dam
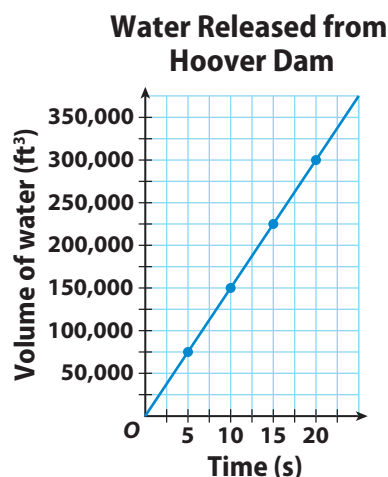
The slope of the line is 15,000. This means that for every second that passed, 15,000 ft³ of water was released from Hoover Dam.

Suppose another dam releases water over the same period of time at a rate of 180,000 ft³ per minute. How do the two rates compare?

180,000 ft³ per minute is equal to 3,000 ft³ per second. This rate is one fifth the rate released by the Hoover Dam over the same time period.

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Representing Proportional Relationships

ESSENTIAL QUESTION

How can you use tables, graphs, and equations to represent proportional situations?

EXPLORE ACTIVITY Real World   FL Prep for 8.EE.2.6

Representing Proportional Relationships with Tables

In 1870, the French writer Jules Verne published *20,000 Leagues Under the Sea*, one of the most popular science fiction novels ever written. One definition of a *league* is a unit of measure equaling 3 miles.

A Complete the table.

Distance (leagues)	1	2	6		20,000
Distance (miles)	3			36	
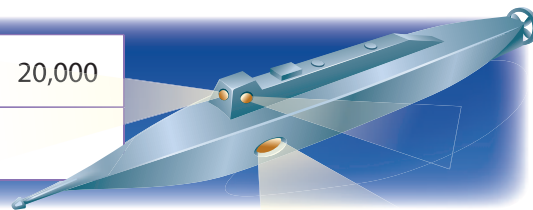
B What relationships do you see among the numbers in the table?

_____

_____

C For each column of the table, find the ratio of the distance in miles to the distance in leagues. Write each ratio in simplest form.

$$\frac{3}{1} = \boxed{} \qquad \frac{\boxed{}}{2} = \boxed{} \qquad \frac{\boxed{}}{6} = \boxed{} \qquad \frac{36}{\boxed{}} = \boxed{} \qquad \frac{\boxed{}}{20,000} = \boxed{}$$

D What do you notice about the ratios? _____

Reflect

1. If you know the distance between two points in leagues, how can you find the distance in miles? _____

2. If you know the distance between two points in miles, how can you find the distance in leagues? _____

Representing Proportional Relationships with Equations

The ratio of the distance in miles to the distance in leagues is constant. This relationship is said to be *proportional*. A **proportional relationship** is a relationship between two quantities in which the ratio of one quantity to the other quantity is constant.

A proportional relationship can be described by an equation of the form $y = kx$, where k is a number called the **constant of proportionality**.

Sometimes it is useful to use another form of the equation, $k = \frac{y}{x}$.

EXAMPLE 1   Real World   FL   8.EE.2.6

Meghan earns $12 an hour at her part-time job. Show that the relationship between the amount she earned and the number of hours she worked is a proportional relationship. Then write an equation for the relationship.

> For every hour Meghan works, she earns $12. So, for 8 hours of work, she earns 8 × $12 = $96.

STEP 1   Make a table relating amount earned to number of hours.

Number of hours	1	2	4	8
Amount earned ($)	12	24	48	96

STEP 2   For each number of hours, write the relationship of the amount earned and the number of hours as a ratio in simplest form.

$\frac{\text{amount earned}}{\text{number of hours}}$ $\frac{12}{1} = \frac{12}{1}$ $\frac{24}{2} = \frac{12}{1}$ $\frac{48}{4} = \frac{12}{1}$ $\frac{96}{8} = \frac{12}{1}$

Since the ratios for the two quantities are all equal to $\frac{12}{1}$, the relationship is proportional.

STEP 3   Write an equation.

> First tell what the variables represent.

Let x represent the number of hours.
Let y represent the amount earned.

Use the ratio as the constant of proportionality in the equation $y = kx$.

The equation is $y = \frac{12}{1}x$ or $y = 12x$.

YOUR TURN

3. Fifteen bicycles are produced each hour at the Speedy Bike Works. Show that the relationship between the number of bikes produced and the number of hours is a proportional relationship. Then write an equation for the relationship. _____

Representing Proportional Relationships with Graphs

You can represent a proportional relationship with a graph. The graph will be a line that passes through the origin (0, 0). The graph shows the relationship between distance measured in miles to distance measured in leagues.
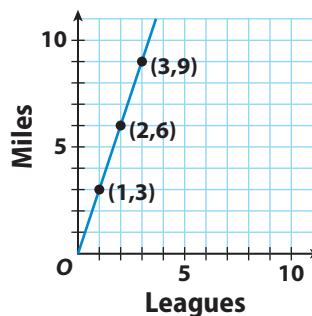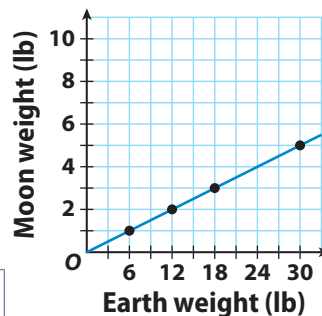
EXAMPLE 2

FL 8.EE.2.6

The graph shows the relationship between the weight of an object on the Moon and its weight on Earth. Write an equation for this relationship.

STEP 1   Use the points on the graph to make a table.

Earth weight (lb)	6	12	18	30
Moon weight (lb)	1	2	3	5

STEP 2   Find the constant of proportionality.

$$\frac{\text{Moon weight}}{\text{Earth weight}} \qquad \frac{1}{6} = \frac{1}{6} \qquad \frac{2}{12} = \frac{1}{6} \qquad \frac{3}{18} = \frac{1}{6} \qquad \frac{5}{30} = \frac{1}{6}$$

The constant of proportionality is $\frac{1}{6}$.

STEP 3   Write an equation.

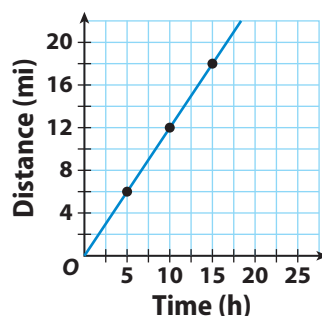Let x represent weight on Earth.

Let y represent weight on the Moon.

The equation is $y = \frac{1}{6}x$.   *Replace k with $\frac{1}{6}$ in $y = kx$.*

YOUR TURN

The graph shows the relationship between the amount of time that a backpacker hikes and the distance traveled.

4. What does the point (5, 6) represent?

_____

5. What is the equation of the relationship?

_____

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Lesson 3.1   **73**

1. **Vocabulary** A proportional relationship is a relationship between two quantities in which the ratio of one quantity to the other quantity

   | is / is not | constant.

2. **Vocabulary** When writing an equation of a proportional relationship in the

   form $y = kx$, k represents the _____.

3. Write an equation that describes the proportional relationship between the number of days and the number of weeks in a given length of time. (Explore Activity and Example 1)

   a. Complete the table.

 | Time (weeks) | 1 | 2 | 4 | | 10 |
   |---|---|---|---|---|---|
   | Time (days) | 7 | | | 56 | |

   b. Let x represent _____.

   Let y represent _____.

   The equation that describes the relationship is _____.

Each table or graph represents a proportional relationship. Write an equation that describes the relationship. (Example 1 and Example 2)

4. **Physical Science** The relationship between the numbers of oxygen atoms and hydrogen atoms in water

Oxygen atoms	2	5		120
Hydrogen atoms	4		34	

   _____

5.

   Map of Iowa

   _____
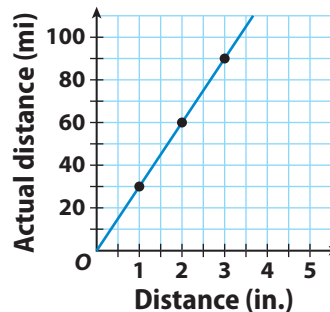
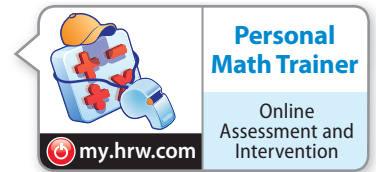 **ESSENTIAL QUESTION CHECK-IN**

6. If you know the equation of a proportional relationship, how can you draw the graph of the equation?

   _____

   _____

3.1 Independent Practice

FL 8.EE.2.6, 8.F.2.4

The table shows the relationship between temperatures measured on the Celsius and Fahrenheit scales.

Celsius temperature	0	10	20	30	40	50
Fahrenheit temperature	32	50	68	86	104	122

7. Is the relationship between the temperature scales proportional? Why or why not?

_____

_____

8. Describe the graph of the Celsius-Fahrenheit relationship.

_____

_____

9. Analyze Relationships Ralph opened a savings account with a deposit of $100. Every month after that, he deposited $20 more.

a. Why is the relationship described not proportional?

_____

_____

b. How could the situation be changed to make the situation proportional?

_____

_____

10. Represent Real-World Problems Describe a real-world situation that can be modeled by the equation $y = \frac{1}{20}x$. Be sure to describe what each variable represents.
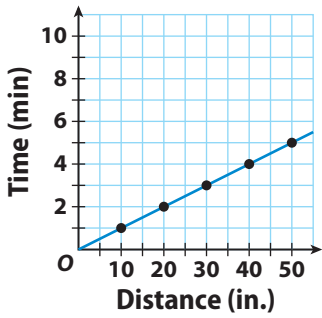
_____

_____

Look for a Pattern **The variables *x* and *y* are related proportionally.**

11. When $x = 8$, $y = 20$. Find y when $x = 42$. _____

12. When $x = 12$, $y = 8$. Find x when $y = 12$. _____

13. The graph shows the relationship between the distance that a snail crawls and the time that it crawls.

Snail Crawling

   a. Use the points on the graph to make a table.

Distance (in.)					
Time (min)					

   b. Write the equation for the relationship and tell what each variable represents.

_____

_____

   c. How long does it take the snail to crawl 85 inches? _____

H.O.T.   **FOCUS ON HIGHER ORDER THINKING**

14. Communicate Mathematical Ideas Explain why all of the graphs in this lesson show the first quadrant but omit the other three quadrants.

_____

_____

_____

_____

15. Analyze Relationships Complete the table.

Length of side of square	1	2	3	4	5
Perimeter of square					
Area of square					

   a. Are the length of a side of a square and the perimeter of the square related proportionally? Why or why not?

_____

_____

   b. Are the length of a side of a square and the area of the square related proportionally? Why or why not?

_____

_____

16. Make a Conjecture A table shows a proportional relationship where k is the constant of proportionality. The rows are then switched. How does the new constant of proportionality relate to the original one?

_____

Rate of Change and Slope

? ESSENTIAL QUESTION

How do you find a rate of change or a slope?

Investigating Rates of Change

A **rate of change** is a ratio of the amount of change in the dependent variable, or *output*, to the amount of change in the independent variable, or *input*.

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EXAMPLE 1 Real World

FL 8.F.2.4

Eve keeps a record of the number of lawns she has mowed and the money she has earned. Tell whether the rates of change are constant or variable.

	Day 1	Day 2	Day 3	Day 4
Number of lawns	1	3	6	8
Amount earned ($)	15	45	90	120

STEP 1 Identify the input and output variables.

Input variable: number of lawns   Output variable: amount earned

STEP 2 Find the rates of change.

Day 1 to Day 2: $\dfrac{\text{change in \$}}{\text{change in lawns}} = \dfrac{45 - 15}{3 - 1} = \dfrac{30}{2} = 15$

Day 2 to Day 3: $\dfrac{\text{change in \$}}{\text{change in lawns}} = \dfrac{90 - 45}{6 - 3} = \dfrac{45}{3} = 15$

Day 3 to Day 4: $\dfrac{\text{change in \$}}{\text{change in lawns}} = \dfrac{120 - 90}{8 - 6} = \dfrac{30}{2} = 15$

The rates of change are constant: $15 per lawn.

Math Talk
Mathematical Practices

Would you expect the rates of change of a car's speed during a drive through a city to be constant or variable? Explain.

YOUR TURN

1. The table shows the approximate height of a football after it is kicked. Tell whether the rates of change are constant or variable.

Time (s)	Height (ft)
0	0
0.5	18
1.5	31
2	26

Find the rates of change:

_____

The rates of change are   constant / variable.

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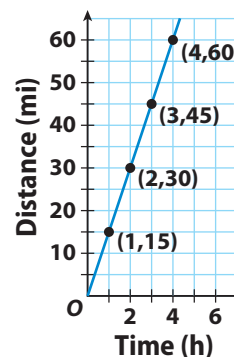
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Using Graphs to Find Rates of Change

You can also use a graph to find rates of change.

The graph shows the distance Nathan bicycled over time.
What is Nathan's rate of change?

A Find the rate of change from 1 hour to 2 hours.

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{30 - \boxed{}}{2 - 1} = \frac{\boxed{}}{1} = \boxed{} \text{ miles per hour}$$

B Find the rate of change from 1 hour to 4 hours.

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{60 - \boxed{}}{4 - \boxed{}} = \frac{\boxed{}}{\boxed{}} = \boxed{} \text{ miles per hour}$$

C Find the rate of change from 2 hour to 4 hours.

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{60 - \boxed{}}{4 - \boxed{}} = \frac{\boxed{}}{\boxed{}} = \boxed{} \text{ miles per hour}$$

D Recall that the graph of a proportional relationship is a line through the origin. Explain whether the relationship between Nathan's time and distance is a proportional relationship.

_____

_____

Reflect

2. **Make a Conjecture** Does a proportional relationship have a constant rate of change?

_____

3. Does it matter what interval you use when you find the rate of change of a proportional relationship? Explain.

_____

_____

The graph shows points: (1,15), (2,30), (3,45), (4,60). Distance (mi) on vertical axis, Time (h) on horizontal axis.

Calculating Slope *m*

When the rate of change of a relationship is constant, any segment of its graph has the same steepness. The constant rate of change is called the *slope* of the line.
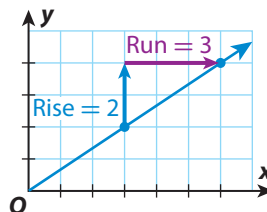
Slope Formula

The **slope** of a line is the ratio of the change in *y*-values (rise) for a segment of the graph to the corresponding change in *x*-values (run).
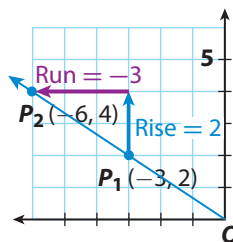
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

EXAMPLE 2

FL   8.F.2.4

Find *m*, the slope of the line.

My Notes

STEP 1 Choose two points on the line.
$P_1(x_1, y_1) = (-3, 2) \quad P_2(x_2, y_2) = (-6, 4)$

STEP 2 Find the change in *y*-values (rise $= y_2 - y_1$) and the change in *x*-values (run $= x_2 - x_1$) as you move from one point to the other.

$\text{rise} = y_2 - y_1$	$\text{run} = x_2 - x_1$
$= 4 - 2$	$= -6 - (-3)$
$= 2$	$= -3$

> If you move up or right, the change is positive. If you move down or left, the change is negative.

STEP 3 $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$
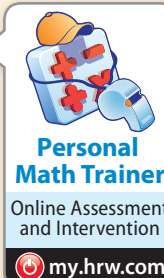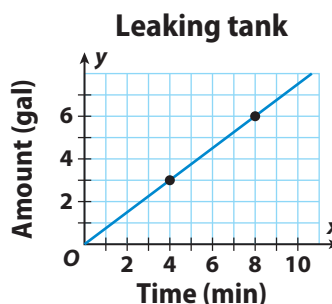
$$= \frac{2}{-3}$$

$$= -\frac{2}{3}$$

YOUR TURN

4. The graph shows the rate at which water is leaking from a tank. The slope of the line gives the leaking rate in gallons per minute. Find the slope of the line.

Leaking tank

Rise = _____   Run = _____

Slope = _____

Tell whether the rates of change are constant or variable. (Example 1)

1. building measurements _____

Feet	3	12	27	75
Yards	1	4	9	25

2. computers sold _____

Week	2	4	9	20
Number Sold	6	12	25	60

3. distance an object falls _____

Distance (ft)	16	64	144	256
Time (s)	1	2	3	4

4. cost of sweaters _____

Number	2	4	7	9
Cost ($)	38	76	133	171

Erica walks to her friend Philip's house. The graph shows Erica's distance from home over time. (Explore Activity)

5. Find the rate of change from 1 minute to 2 minutes.

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{400 - \boxed{}}{2 - \boxed{}} = \frac{\boxed{}}{\boxed{}} = \boxed{} \text{ ft per min}$$
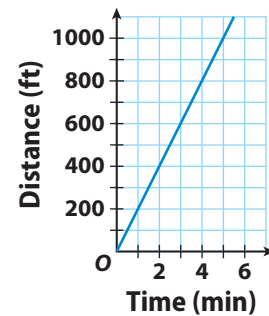
6. Find the rate of change from 1 minute to 4 minutes. _____

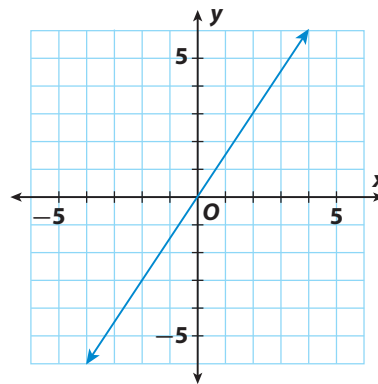**Find the slope of each line.** (Example 2)
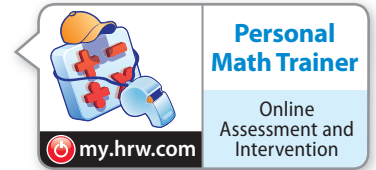
7.

slope = _____

8.
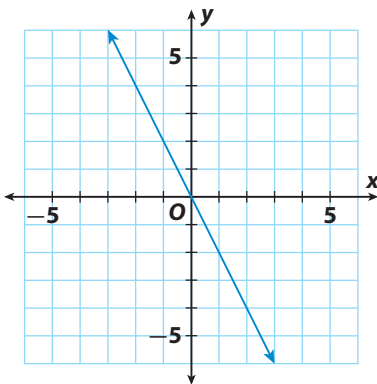
slope = _____

9. If you know two points on a line, how can you find the rate of change of the variables being graphed?

_____

_____

_____

3.2 Independent Practice

FL 8.F.2.4

10. Rectangle *EFGH* is graphed on a coordinate plane with vertices at *E*(−3, 5), *F*(6, 2), *G*(4, −4), and *H*(−5, −1).

    a. Find the slopes of each side.

    _____

    b. What do you notice about the slopes of opposite sides?

    _____

    c. What do you notice about the slopes of adjacent sides?

    _____

11. A bicyclist started riding at 8:00 A.M. The diagram below shows the distance the bicyclist had traveled at different times. What was the bicyclist's average rate of speed in miles per hour?

8:00 A.M. ← 4.5 miles → 8:18 A.M. ← 7.5 miles → 8:48 A.M.

_____

12. Multistep A line passes through (6, 3), (8, 4), and (*n*, −2). Find the value of *n*.

_____

13. A large container holds 5 gallons of water. It begins leaking at a constant rate. After 10 minutes, the container has 3 gallons of water left.

    a. At what rate is the water leaking?

    _____

    b. After how many minutes will the container be empty?

    _____

**14. Critique Reasoning** Billy found the slope of the line through the points (2, 5) and (−2, −5) using the equation $\frac{2-(-2)}{5-(-5)} = \frac{2}{5}$. What mistake did he make?

_____

_____

15. Multiple Representations Graph parallelogram *ABCD* on a coordinate plane with vertices at *A*(3, 4), *B*(6, 1), *C*(0, −2), and *D*(−3, 1).

a. Find the slope of each side.

_____

_____

b. What do you notice about the slopes?

_____

_____

c. Draw another parallelogram on the coordinate plane. Do the slopes have the same characteristics?

_____

H.O.T.   **FOCUS ON HIGHER ORDER THINKING**

Work Area
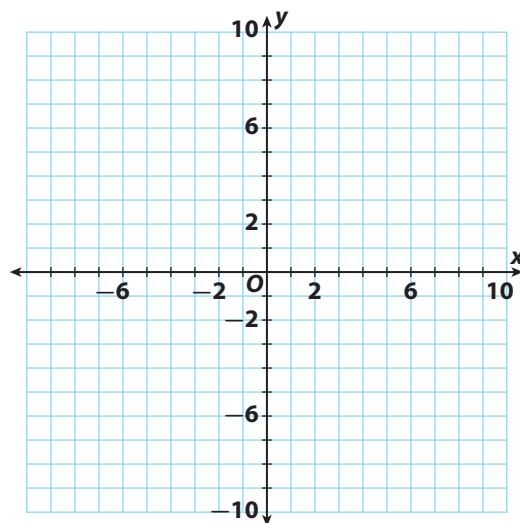
16. Communicate Mathematical Ideas Ben and Phoebe are finding the slope of a line. Ben chose two points on the line and used them to find the slope. Phoebe used two different points to find the slope. Did they get the same answer? Explain.

_____

_____

_____

17. Analyze Relationships Two lines pass through the origin. The lines have slopes that are opposites. Compare and contrast the lines.

_____

_____

_____

_____

_____

18. Reason Abstractly What is the slope of the *x*-axis? Explain.

_____

_____

_____

Interpreting the Unit Rate as Slope

? ESSENTIAL QUESTION

How do you interpret the unit rate as slope?

EXPLORE ACTIVITY Real World

FL 8.EE.2.5, 8.F.2.4

Relating the Unit Rate to Slope

A rate is a comparison of two quantities that have different units, such as miles and hours. A **unit rate** is a rate in which the second quantity in the comparison is one unit.

A storm is raging on Misty Mountain. The graph shows the constant rate of change of the snow level on the mountain.

Misty Mountain Storm
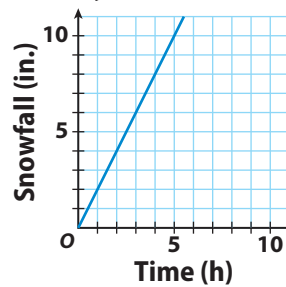
A Find the slope of the graph using the points (1, 2) and (5, 10). Remember that the slope is the constant rate of change.

B Find the unit rate of snowfall in inches per hour. Explain your method.

_____

_____

C Compare the slope of the graph and the unit rate of change in the snow level. What do you notice?

_____

D Which unique point on this graph gives you the slope of the graph and the unit rate of change in the snow level? Explain how you found the point.

_____

_____

_____

_____

Graphing Proportional Relationships

You can use a table and a graph to find the unit rate and slope that describe a real-world proportional relationship. The constant of proportionality for a proportional relationship is the same as the slope.

EXAMPLE 1 · Real World

FL · 8.EE.2.5

Every 3 seconds, 4 cubic feet of water pass over a dam. Draw a graph of the situation. Find the unit rate of this proportional relationship.
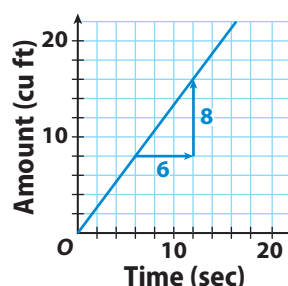
STEP 1 Make a table.

Time (s)	3	6	9	12	15
Volume (ft³)	4	8	12	16	20

STEP 2 Draw a graph.

STEP 3 Find the slope.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{8}{6}$$

$$= \frac{4}{3}$$

Water Over the Dam

The unit rate of water passing over the dam and the slope of the graph of the relationship are equal, $\frac{4}{3}$ cubic feet per second.

Math Talk
Mathematical Practices

In a proportional relationship, how are the constant of proportionality, the unit rate, and the slope of the graph of the relationship related?

Reflect

1. **What If?** Without referring to the graph, how do you know that the point $\left(1, \frac{4}{3}\right)$ is on the graph?

_____

_____

YOUR TURN

2. Tomas rides his bike at a steady rate of 2 miles every 10 minutes. Graph the situation. Find the unit rate of this proportional relationship.

_____

_____

_____

Tomas's Ride
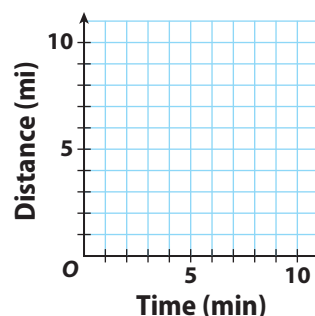
Using Slopes to Compare Unit Rates

You can compare proportional relationships presented in different ways.
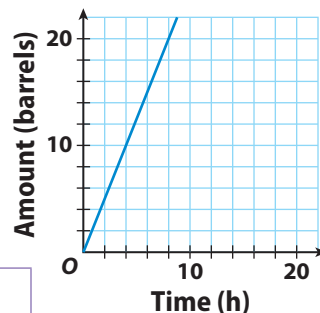
EXAMPLE 2

FL 8.EE.2.5, 8.F.1.2

The equation $y = 2.75x$ represents the rate, in barrels per hour, that oil is pumped from Well A. The graph represents the rate that oil is pumped from Well B. Which well pumped oil at a faster rate?

Well B Pumping Rate

STEP 1 Use the equation $y = 2.75x$ to make a table for Well A's pumping rate, in barrels per hour.

Time (h)	1	2	3	4
Quantity (barrels)	2.75	5.5	8.25	11

STEP 2 Use the table to find the slope of the graph of Well A.

$$\text{slope} = \text{unit rate} = \frac{5.5 - 2.75}{2 - 1} = \frac{2.75}{1} = \textbf{2.75 barrels/hour}$$

STEP 3 Use the graph to find the slope of the graph of Well B.

$$\text{slope} = \text{unit rate} = \frac{\text{rise}}{\text{run}} = \frac{10}{4} = \textbf{2.5 barrels/hour}$$

STEP 4 Compare the unit rates.

$2.75 > 2.5$, so Well A's rate, 2.75 barrels/hour, is faster.

Reflect

3. Describe the relationships among the slope of the graph of Well A's rate, the equation representing Well A's rate, and the constant of proportionality.

_____

_____

YOUR TURN

4. The equation $y = 375x$ represents the relationship between x, the time that a plane flies in hours, and y, the distance the plane flies in miles for Plane A. The table represents the relationship for Plane B. Find the slope of the graph for each plane and the plane's rate of speed. Determine which plane is flying at a faster rate of speed.

Time (h)	1	2	3	4
Distance (mi)	425	850	1275	1700

_____

Personal Math Trainer
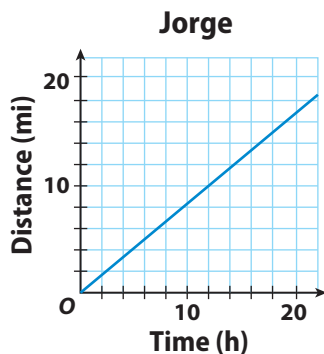
Online Assessment and Intervention

my.hrw.com

Give the slope of the graph and the unit rate. (Explore Activity and Example 1)
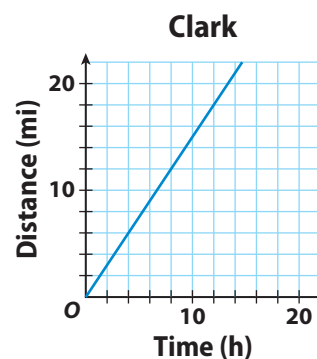
1. Jorge: 5 miles every 6 hours

Jorge

_____

2. Akiko

Time (h)	4	8	12	16
Distance (mi)	5	10	15	20

Akiko

_____
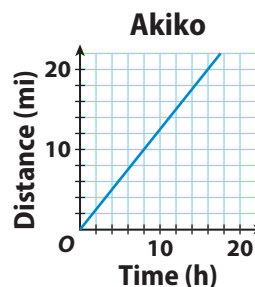
3. The equation $y = 0.5x$ represents the distance Henry hikes, in miles, over time, in hours. The graph represents the rate that Clark hikes. Determine which hiker is faster. Explain. (Example 2)

_____

_____

_____

Clark

Write an equation relating the variables in each table. (Example 2)

4.

Time (x)	1	2	4	6
Distance (y)	15	30	60	90

_____

5.

Time (x)	16	32	48	64
Distance (y)	6	12	18	24

_____

? ESSENTIAL QUESTION CHECK-IN

6. Describe methods you can use to show a proportional relationship between two variables, x and y. For each method, explain how you can find the unit rate and the slope.

_____

_____

_____

_____

_____

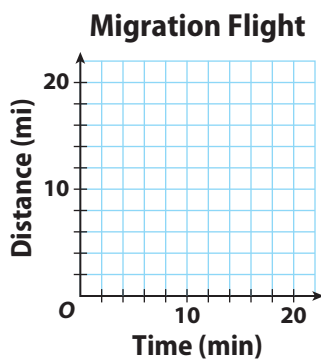# 3.3 Independent Practice

FL 8.EE.2.5, 8.F.1.2, 8.F.2.4

7. A Canadian goose migrated at a steady rate of 3 miles every 4 minutes.

    a. Fill in the table to describe the relationship.

Time (min)	4	8			20
Distance (mi)			9	12	

    b. Graph the relationship.

Migration Flight

    c. Find the slope of the graph and describe what it means in the context of this problem.

    _____

    _____

    _____

    _____

8. **Vocabulary** A unit rate is a rate in which the
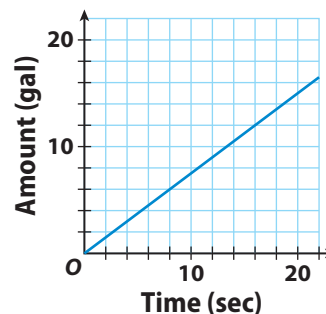
    first quantity / second quantity in the comparison is one unit.

9. The table and the graph represent the rate at which two machines are bottling milk in gallons per second.

Machine 1

Time (s)	1	2	3	4
Amount (gal)	0.6	1.2	1.8	2.4

Machine 2

    a. Determine the slope and unit rate of each machine.

    _____

    _____

    b. Determine which machine is working at a faster rate.

    _____

10. Cycling The equation $y = \frac{1}{9}x$ represents the distance y, in kilometers, that Patrick traveled in x minutes while training for the cycling portion of a triathlon. The table shows the distance y Jennifer traveled in x minutes in her training. Who has the faster training rate?

Time (min)	40	64	80	96
Distance (km)	5	8	10	12

_____

_____

_____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

11. Analyze Relationships There is a proportional relationship between minutes and dollars per minute, shown on a graph of printing expenses. The graph passes through the point (1, 4.75). What is the slope of the graph? What is the unit rate? Explain.

_____

_____

_____

12. Draw Conclusions Two cars start at the same time and travel at different constant rates. A graph for Car A passes through the point (0.5, 27.5), and a graph for Car B passes through (4, 240). Both graphs show distance in miles and time in hours. Which car is traveling faster? Explain.

_____

_____

_____

_____

13. Critical Thinking The table shows the rate at which water is being pumped into a swimming pool.

Time (min)	2	5	7	12
Amount (gal)	36	90	126	216

Use the unit rate and the amount of water pumped after 12 minutes to find how much water will have been pumped into the pool after $13\frac{1}{2}$ minutes. Explain your reasoning.

_____

_____

_____

Ready to Go On?

3.1 Representing Proportional Relationships

1. Find the constant of proportionality for the table of values.
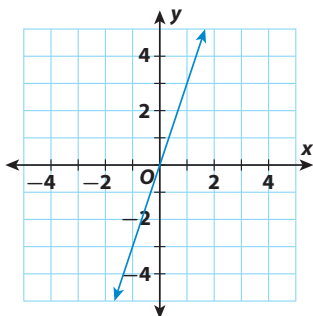
x	2	3	4	5
y	3	4.5	6	7.5

_____

2. Phil is riding his bike. He rides 25 miles in 2 hours, 37.5 miles in 3 hours, and 50 miles in 4 hours. Find the constant of proportionality and write an equation to describe the situation.
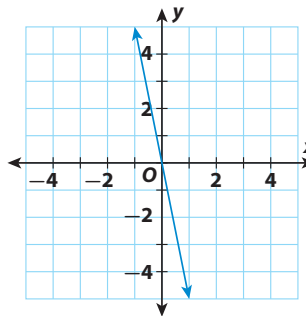
_____

3.2 Rate of Change and Slope

Find the slope of each line.

3.

_____

4.

_____

3.3 Interpreting the Unit Rate as Slope

5. The distance Train A travels is represented by $d = 70t$, where d is the distance in kilometers and t is the time in hours. The distance Train B travels at various times is shown in the table. What is the unit rate of each train? Which train is going faster?

Time (hours)	Distance (km)
2	150
4	300
5	375

_____

? ESSENTIAL QUESTION

6. What is the relationship among proportional relationships, lines, rates of change, and slope?

_____

_____

Selected Response

1. Which of the following is equivalent to 5^{-1}?

Ⓐ 4
Ⓒ $-\frac{1}{5}$
Ⓑ $\frac{1}{5}$
Ⓓ -5

2. Prasert earns $9 an hour. Which table represents this proportional relationship?

Ⓐ
Hours	4	6	8
Earnings ($)	36	54	72

Ⓑ
Hours	4	6	8
Earnings ($)	36	45	54

Ⓒ
Hours	2	3	4
Earnings ($)	9	18	27

Ⓓ
Hours	2	3	4
Earnings ($)	18	27	54

3. A factory produces widgets at a constant rate. After 4 hours, 3,120 widgets have been produced. At what rate are the widgets being produced?

Ⓐ 630 widgets per hour

Ⓑ 708 widgets per hour

Ⓒ 780 widgets per hour

Ⓓ 1,365 widgets per hour

4. A full lake begins dropping at a constant rate. After 4 weeks it has dropped 3 feet. What is the unit rate of change in the lake's level compared to its full level?

Ⓐ 0.75 feet per week

Ⓑ 1.33 feet per week

Ⓒ −0.75 feet per week

Ⓓ −1.33 feet per week

5. What is the slope of the line below?
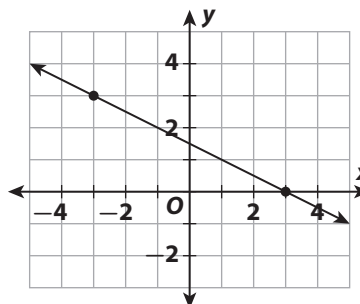
Ⓐ −2
Ⓒ $\frac{1}{2}$
Ⓑ $-\frac{1}{2}$
Ⓓ 2

6. Jim earns $41.25 in 5 hours. Susan earns $30.00 in 4 hours. Pierre's hourly rate is less than Jim's, but more than Susan's. What is his hourly rate?

Ⓐ $6.50
Ⓒ $7.35
Ⓑ $7.75
Ⓓ $8.25

Mini-Task

7. Joelle can read 3 pages in 4 minutes, 4.5 pages in 6 minutes, and 6 pages in 8 minutes.

a. Make a table of the data.

Minutes			
Pages			

b. Use the values in the table to find the unit rate.

_____

c. Graph the relationship between minutes and pages read.

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